Aerodynamic forces and moments on cubes and flat plates, with applications to wind-borne debris

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ABSTRACT: The drag forces on a cube at various angles of combined pitch and yaw were measured in smooth flow. For varying pitch at zero yaw, the results were compared with data published by ESDU and found to agree well. An average drag coefficient to allow for random orientations was modified for the effects of small-scale turbulence, and used in a numerical model of the trajectories of cubes measured in a wind tunnel. Reasonable agreement was obtained. The aerodynamic drag and lift forces and moments acting on a square flat plate are discussed and a model of these as a function of pitch angle, for the calculation of trajectories, is presented. Comparison of the calculated initial trajectories of a flat plate with that obtained experimentally by Tachikawa was made, and good agreement found.

1.0 INTRODUCTION

Although testing of building materials for resistance to impact by wind-borne debris has been carried out for many years, this has been done with little knowledge of the aerodynamics and mechanics of wind-borne debris items. Numerical modeling of the trajectories of generic bluff-body shapes representative of real debris items appears to be the most useful approach to make engineering predictions of missile speeds and trajectories. An essential ingredient of such numerical models is a description of the coefficients of aerodynamic forces and moments as a function of the angles of pitch, roll and yaw with respect to the relative wind vector.

In this paper, the relevant aerodynamic coefficients of cubes and flat plates are discussed, and some comparisons made of calculated trajectories with experimentally measured ones in wind-tunnel experiments.

2.0 REVIEW OF PREVIOUS WORK

In the nineteen-seventies, there was considerable interest in the design of nuclear facilities for the effects of tornadoes, and several numerical models of the trajectories of wind-borne missiles were set up. Although measurements of aerodynamic coefficients for generic debris items were carried out at that time (Marte, Kurtz and Redmann, [1]), this data was not published and is apparently not currently available.

Twisdale, Dunn and Davis [2], and Twisdale, Vickery and Steckley [3], describe models of missile transport based on random orientation for tornadoes and hurricanes, respectively. A cross-flow aerodynamics model for prismatic missiles, based on measured drag coefficients for the orthogonal wind directions, was used to determine all the aerodynamic force coefficients.

Tachikawa [4] used a combination of experimental (wind-tunnel) and numerical simulations to study the trajectories of flat plates and prisms. He measured lift, drag and moment coefficients on auto-rotating flat plates, and measured trajectories for various initial angles of attack. These measurements are used for comparison later in the present paper.

More recently, Wills *et al* [5] studied the conditions of threshold of flight for various generic missile types, classified as ‘compact’, ‘sheets’ and ‘rods’. Wang and Letchford [6] measured wind speeds for the initiation of flight of small ‘sheet’ objects in a wind tunnel, and compared them with the model of Wills *et al*.

Holmes [7] made numerical calculations of the trajectories of spheres in turbulent boundary-layer flow, and studied the effect of free-stream atmospheric turbulence, and vertical air resistance.

3.0 AERODYNAMIC COEFFICIENTS FOR A CUBE AND FLAT PLATE

Although mean aerodynamic force coefficients on bluff bodies are often sensitive to small scale turbulence with length scales comparable to, or smaller than, the body dimension, typical wind-borne debris objects are much smaller than the scales of atmospheric turbulence, and thus smooth-flow aerodynamic coefficients applied in a quasi-steady way should be applicable.

3.1 Drag coefficients for a cube

Drag coefficients for cubes at various angles of pitch are given in ESDU 71016, [8]. ESDU 71016, however, does not give drag coefficients for cubes with combined non-zero pitch and yaw angles. Hence some measurements were carried out in smooth flow in a small wind tunnel (cross-section dimensions : 950mm by 600mm) at Louisiana State University, with cubes of side length of 75mm, and wind speeds up to 30 m/s.

Drag coefficients (normalized by the frontal area for zero pitch and yaw) measured at LSU are shown in Table 1. The values for symmetrical pitch and yaw angles were averaged to give the values in Table 1.

	0^o pitch	15^o pitch	30^o pitch	45^o pitch
0^o yaw	1.00	1.05	1.10	1.16
15^o yaw	1.05	1.15	1.23	1.27
30^o yaw	1.10	1.23	1.22	1.21
45^o yaw	1.16	1.27	1.21	1.21

Table 1: Drag coefficients for a cube

The values for zero degree yaw, with varying pitch, agree well with the values given in ESDU 71016, as shown in Figure 1.

For a randomly oriented cube, the average value from Table I is 1.16. ESDU 71016 also gives a tentative reduction factor of 0.72 for (small-scale) longitudinal turbulence intensities between 7 and 10 %.

3.2 Aerodynamic coefficients for an inclined flat plate

Hoerner [9] (Figure 29) gives the normal force coefficient on a square flat plate at various angles of attack. This can be represented to a good approximation by the function shown in Figure 2.

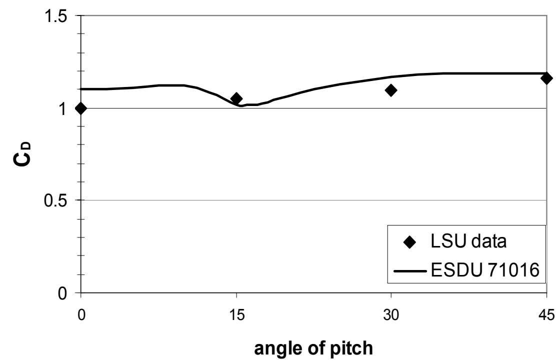

Figure 1: C_D for a cube - comparison of LSU data with ESDU 71016 (smooth flow)

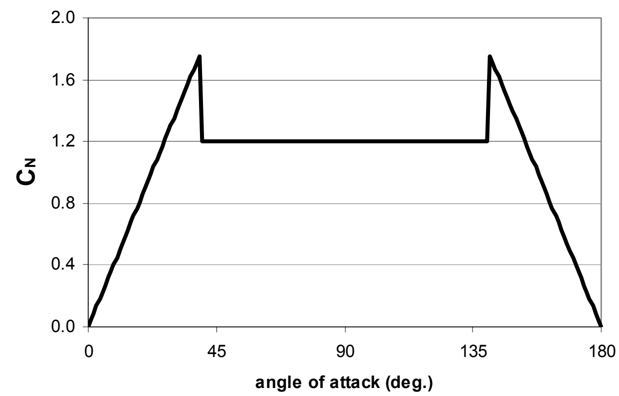

Figure 2: Assumed normal force coefficient for an inclined flat plate (after Hoerner, [9])

The resulting lift and drag forces due to normal pressures on the plate can then be obtained from the resolution of the normal force into components normal and parallel to the wind direction. The pitching moment acting about the centroid of the plate is obtained by multiplying the normal force by the distance of the centre of pressure from the centroid, c. A tentative model of the distance of the centre of pressure, as a fraction of the side dimension of the plate, ℓ, is shown in Figure 3.

For angles of attack between 0 and 20 degrees, the centre of pressure is assumed to be at the quarter chord point ($c/\ell = 0.25$). (c/ℓ) is then assumed to vary linearly between 0.25 and zero, as the angle of attack varies between 20 and 60 degrees. Between 60 and 90 degrees, the centre of pressure is assumed to be at the centre of the plate. The position of the centre of pressure at an angle of attack of 45 degrees ($c/\ell = 0.094$) agrees closely with a value obtained by the first author at Monash University (unpublished).

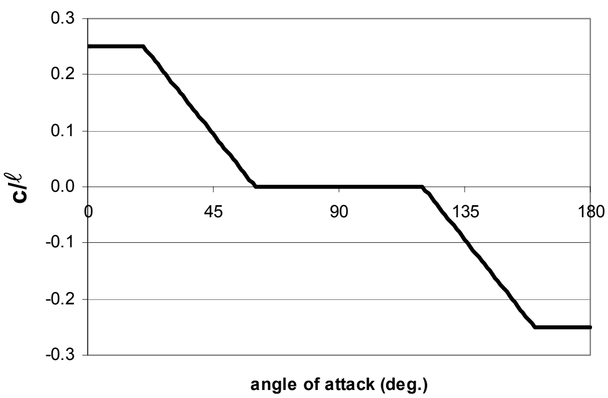

Figure 3: Centre of pressure location for a square plate

4.0 CALCULATED TRAJECTORIES

4.1 Trajectories of cubes

The relative wind velocity vectors with respect to a moving cube in a horizontal wind field are shown in Figure 4. Neglecting lift forces, the horizontal and vertical accelerations can be shown to be given by :

$$\frac{d^2x}{dt^2} = \frac{\rho_a C_D (U - u_m)\sqrt{\left[(U - u_m)^2 + v_m{}^2\right]}}{2\rho_m . \ell} \tag{1}$$
$$= K(U - u_m)\sqrt{\left[(U - u_m)^2 + v_m{}^2\right]}$$

$$\frac{d^2z}{dt^2} = \frac{\rho_a C_D (-v_m)\sqrt{\left[(U - u_m)^2 + v_m{}^2\right]}}{2\rho_m . \ell} - g \tag{2}$$
$$= K(-v_m)\sqrt{\left[(U - u_m)^2 + v_m{}^2\right]} - g$$

where $K = \frac{\rho_a C_D}{2\rho_m \ell}$, ρ_a is the density of air, ρ_m is the density of the cube, and ℓ is the side length of the cube. Equations (1) and (2) can readily be solved numerically for small incremental time steps.

Experimental measurements were made of trajectories of cubes in a wind tunnel at Texas Tech University. The experimental approach is described by Wang and Letchford [6].

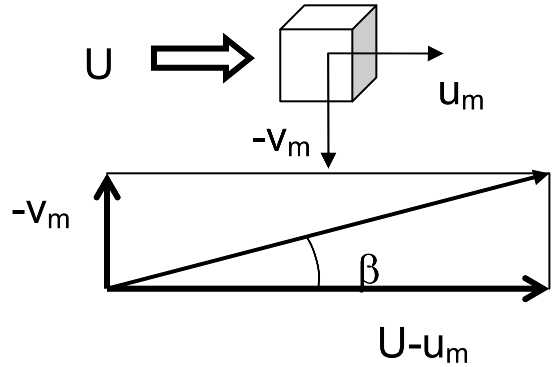

Figure 4: Relative wind vectors with respect to a wind-borne cube

Numerically calculated values of the horizontal displacements at impact with the floor (350 mm below the release point) are compared with the corresponding measured values in Figure 5.

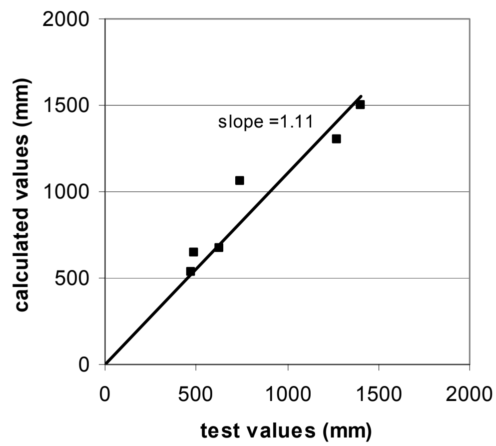

Figure 5: Experimental and numerically calculated displacements for cubes

The agreement is generally good, although the numerical calculations have generally over-estimated the displacements and velocities at impact (comparison of velocities is not shown). This may be because the reduced wind velocity in the wake of the release platform was not accounted for in the calculations; however, the reduced drag coefficient due to small-scale turbulence in the boundary layer near the floor of the wind tunnel was accounted for, using the ESDU reduction factor (see Section 3.1 above).

4.2 *Trajectories of inclined flat plates*

To calculate the trajectories of flat plates, the lift force and pitching moment induced by the relative wind and the rotational motion of the plate are significant. The following equations then describe the horizontal, vertical and angular accelerations of the plate :

$$\frac{d^2x}{dt^2} = \frac{\rho_a (C_D \cos\beta - C_L \sin\beta)\left[(U-u_m)^2 + v_m{}^2\right]}{2\rho_m . t} \qquad (3)$$

$$\frac{d^2z}{dt^2} = \frac{\rho_a (C_D \sin\beta + C_L \cos\beta)\left[(U-u_m)^2 + v_m{}^2\right]}{2\rho_m . t} \qquad (4)$$

$$\frac{d^2\theta}{dt^2} = \frac{\rho_a C_M A \,\ell\left[(U-u_m)^2 + v_m{}^2\right]}{2I} \qquad (5)$$

where β is the angle of attack of the relative wind induced by the vertical motion of the plate, with respect to the horizontal, t is the plate thickness, A is the plan area (ℓ^2), and I is the mass moment of inertia.

Tachikawa [4] measured in a wind tunnel the trajectories of square flat plates released at various initial angles of attack. Calculations were made of these trajectories, using Equations (3) to (5) and the model of aerodynamic coefficients given in Section 3.2. An example of one of these comparisons is given in Figure 6. The agreement is good for all three displacements.

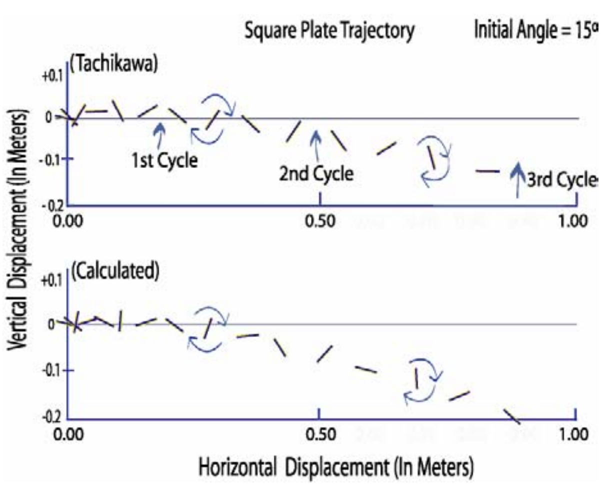

Figure 6: Experimental and numerically calculated displacements for a square plate (measured trajectory by Tachikawa [4] above; calculated trajectory below)

Tachikawa [4] measured the additional lift forces induced by angular rotations (Magnus effect); these were included in the calculations, based on Tachikawa's measurements of forces on auto-rotating plates. The effects on the vertical displacement were found to be significant.

5.0 CONCLUSIONS

The aerodynamic coefficients of cubes and flat plates, representative of generic wind-borne debris have been discussed. Experimentally measured trajectories in wind tunnels for these two shapes are compared with numerical calculations. The agreement is generally good.

6.0 ACKNOWLEDGEMENTS

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7.0 REFERENCES

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